PHYS 100C, MIDTERM SOLUTIONS

Tuesday, May 12, 2009
12:56 PM

*9.2

$$f = A\sin(kz)\cdot\cos(kvt)$$

$$\frac{\partial f}{\partial z} = A\cdot k\cdot\cos(kz)\cdot\cos(kvt)$$

$$\frac{\partial^2 f}{\partial z^2} = -Ak^2\cdot\sin(kz)\cdot\cos(kvt) = -k^2\cdot f$$

$$\frac{\partial f}{\partial t} = -A\cdot kv\cdot\sin(kz)\cdot\sin(kvt)$$

$$\frac{\partial^2 f}{\partial t^2} = -A(kv)^2\sin(kz)\cdot\cos(kvt) = -(kv)^2 f$$

$$\frac{\partial^2 f}{\partial t^2} = -(kv)^2\cdot f = v^2\left(-k^2\cdot f\right) = v^2\cdot\frac{\partial^2 f}{\partial z^2}$$

$$\sin A\cdot\cos B = \frac{1}{2}\left(\sin(A+B)+\sin(A-B)\right)$$

(Why? Remember that

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta+\sin\beta\cdot\cos\alpha$$
$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta-\sin\beta\cdot\cos\alpha$$

Add the two and $\cdot\frac{1}{2}$)

$$f = \frac{A}{2}\left[\underbrace{\sin(kz+kvt)}_{\text{wave travels in negative } z}+\underbrace{\sin(kz-kvt)}_{\text{wave travels in positive } z}\right]$$

*9.25

$$k = \frac{\omega}{c}\left[1+\frac{Nq^2}{2m\varepsilon_0}\sum\frac{f_i}{\omega_i^2-\omega^2}\right]$$

Phase velocity:

$$V_p = \frac{\omega}{k} = \frac{c}{1+\frac{Nq^2}{2m\varepsilon_0}\sum\frac{f_i}{\omega_i^2-\omega^2}}$$

$V_p > c$ for $\omega \gg \omega_i$

$V_p < c$ for $\omega \ll \omega_i$

Group velocity:

$$V_g = \frac{\partial\omega}{\partial k} = \left(\frac{\partial k}{\partial\omega}\right)^{-1}$$

$$\frac{\partial k}{\partial \omega} = \frac{1}{c}\left[1 + \frac{Nq^2}{2m\varepsilon_0}\left(\sum \frac{f_j}{\omega_j^2-\omega^2} + \omega \sum \frac{f_j \cdot 2\omega}{(\omega_j^2-\omega^2)^2}\right)\right] =$$

$$= \frac{1}{c}\left(1 + \frac{Nq^2}{2m\varepsilon_0} \sum f_j \frac{\omega_j^2+\omega^2}{(\omega_j^2-\omega^2)^2}\right)$$

Therefore

$$v_g = \left(\frac{\partial k}{\partial \omega}\right)^{-1} = \frac{c}{1 + \frac{Nq^2}{2m\varepsilon_0} \sum f_j \frac{(\omega_j^2+\omega^2)}{(\omega_j^2-\omega^2)^2}} < c$$

Since $\frac{\omega_j^2+\omega^2}{(\omega_j^2-\omega^2)^2} > 0$

*** 10.4** $V = 0$

$$A = A_0 \cdot \sin(kx-\omega t)\,\hat{y}$$

$$E = -\nabla V - \frac{\partial A}{\partial t} = 0 - A_0(-\omega)\cdot\cos(kx-\omega t)\,\hat{y} =$$

$$= A_0 \cdot \omega \cdot \cos(kx-\omega t)\cdot\hat{y}$$

$$B = \nabla \times A = \hat{z}\cdot\frac{\partial}{\partial x} A_y = A_0 k \cdot \cos(kx-\omega t)\cdot\hat{z}$$

(all other components of $\nabla \times A$ are 0)

* $\nabla \cdot E = 0$ since $\frac{\partial E_y}{\partial y} = 0$

* $\nabla \cdot B = 0$ since $\frac{\partial B_z}{\partial z} = 0$

$$\nabla \times E = \hat{z}\cdot\frac{\partial E_y}{\partial x} = -A_0 \cdot \omega k \cdot \sin(kx-\omega t)\,\hat{z}$$

$$-\frac{\partial B}{\partial t} = -A_0 \cdot k \cdot (-\omega)\cdot(-\sin(kx-\omega t))\cdot\hat{z} =$$

$$= -A_0 \omega k \cdot \sin(kx-\omega t)\cdot\hat{z}$$

* $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\nabla \times B = -\hat{y} \cdot \frac{\partial B_z}{\partial x} = A_0 \cdot k^2 \cdot \sin(kx - \omega t) \cdot \hat{y}$$

(all other terms in $\nabla \times B$ are 0)

$$\frac{\partial E}{\partial t} = A_0 \cdot \omega^2 \cdot \sin(kx - \omega t) \cdot \hat{y}$$

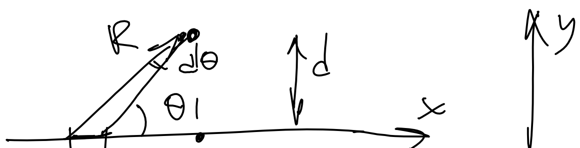
So $\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$

ONLY if $k^2 = \mu_0 \varepsilon_0 \cdot \omega^2$

OR $c \equiv \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

* 10.19

$$E = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1 - v^2/c^2}{\left(1 - \frac{v^2 \sin^2\theta}{c^2}\right)^{3/2}} \cdot \frac{\hat{R}}{R^2}$$



$x = \frac{d}{\sin\theta}$

$dx = \frac{d \cdot d\theta}{\sin^2\theta}$

Charge in dx: $dq = \lambda \cdot dx = \frac{\lambda d \cdot d\theta}{\sin^2\theta}$

Out of symmetry, E points perpendicular to x ($\vec{E} \parallel \hat{y}$), $\vec{E}_x = 0$ since contributions to E_x from $+\theta$ and $-\theta$ cancel.

$$dE_y = \frac{dq \cdot \sin\theta}{4\pi\varepsilon_0} \cdot \frac{\left(1 - v^2/c^2\right)}{\left(1 - \frac{v^2 \cdot \sin^2\theta}{c^2}\right)^{3/2}} \cdot \frac{1}{R^2}$$

Replace $R = \frac{d}{\sin\theta}$ and integrate dE_y from $\theta_1 = 0$ to $\theta_2 = \pi$:

$$dE_y = \frac{\lambda \cancel{d} \sin\theta \cdot d\theta}{4\pi\varepsilon_0 \cdot \cancel{\sin^2\theta}} \cdot \frac{\left(1-v^2/c^2\right)}{\left(1-\frac{v^2 \cdot \sin^2\theta}{c^2}\right)^{3/2}} \cdot \frac{\cancel{\sin^2\theta}}{d^{\cancel{2}}}$$

$$E_y = \int dE_y = \frac{\lambda\left(1-v^2/c^2\right)}{4\pi\varepsilon_0 \cdot d} \int_0^\pi \frac{\sin\theta \cdot d\theta}{\left(b + a\cdot\cos^2\theta\right)^{3/2}}$$

where $a = \frac{v^2}{c^2}$ $b = 1 - v^2/c^2$

(we replaced $\sin^2\theta = 1-\cos^2\theta$),
Note that $\sin\theta \cdot d\theta = d(\cos\theta)$
and using our hint:

$$-\int_0^\pi \frac{d(\cos\theta)}{b^{3/2}\left(1+\frac{a}{b}\cdot\cos^2\theta\right)^{3/2}} = -\frac{\cos\theta}{b^{3/2}\left(1+\frac{a}{b}\cdot\cos^2\theta\right)^{1/2}}\Bigg|_0^\pi =$$

$$= \frac{2}{b^{3/2}\left(1+\frac{a}{b}\right)^{1/2}} = \frac{2}{b\,(b+a)^{1/2}} = \frac{2}{\left(1-\frac{v^2}{c^2}\right)}$$

$$E_y = \frac{\lambda \cancel{\left(1-\frac{v^2}{c^2}\right)}}{4\pi\varepsilon_0 \cdot d} \cdot \frac{2}{\cancel{\left(1-\frac{v^2}{c^2}\right)}} = \frac{\lambda}{2\pi\varepsilon_0 d}$$, OR

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 d} \cdot \hat{y}$$, SAME as for static charged wire! (Gauss' Law)

$$\vec{B} = \frac{1}{c^2}\left(\vec{v} \times \vec{E}\right) = \frac{v}{c^2}\left(\hat{x} \times \left(\int dE_y\right)\hat{y}\right) = \frac{v E_y}{c} \cdot \hat{z}$$

OR $$\vec{B} = \frac{\lambda v}{2\pi\varepsilon_0 d \cdot c^2} \cdot \hat{z} = \frac{\mu_0}{2\pi} \cdot \frac{\lambda \cdot v}{d} \cdot \hat{z}$$

(used $\frac{1}{\varepsilon_0 c^2} = \mu_0$) $\uparrow$ AKA Ampere's Law